Patronage of Rector
of University of Lodz

FACULTY OF MATHEMATICS
AND COMPUTER SCIENCE
University of Lodz

Dynamics of (Semi-)Group Actions

June 26-28, 2023, Łódź, Poland

Program & Abstracts

Dynamics of (Semi-)Group Actions
June 26-28, 2023, Łódź, Poland

Main speakers

- **Maria Carvalho** University of Porto, Portugal
- **Udajan Darji** University of Louisville, USA
- **Tushar Das** University of Wisconsin- La Crosse, USA
- **Johannes Jaerisch** Nagoya University, Japan
- **Dongkui Ma** South China University of Technology, China
- **Eugen Mihailescu** Romanian Academy, Romania
- **Giulio Tiozzo** University of Toronto, Canada
- **Benjamin Weiss** The Hebrew University of Jerusalem, Israel

Scientific Committee

- **Andrzej Biś** University of Lodz, Poland
- **Dikran Dikranjan** University of Udine, Italy
- **Tomasz Downarowicz** Wrocław University of Science and Technology, Poland
- **L'ubomír Snoha** Matej Bel University, Slovakia
- **Luchezar Stoyanov** University of Western Australia, Australia
- **Mariusz Urbański** University of North Texas, USA
- **Paulo Varandas** Federal University of Bahia, Brazil; University of Porto, Portugal

Local organizers

- **Andrzej Biś** University of Lodz - chairman
- **Kamil Niedziałomski** University of Lodz - secretary
- **Marek Badura** University of Lodz
- **Adam Bartoszek** University of Lodz
- **Małgorzata Ciska-Niedziałomska** University of Lodz
- **Marzena Jaworska-Banert** University of Lodz
- **Wojciech Kozłowski** University of Lodz
- **Ryszard Pawlak** University of Lodz

List of talks

Invited talks

Maria Carvalho	*Historic behavior within semigroup actions*
Udajan Darji	*Odometers and Solenoids*
Tushar Das	*Dimension theory of conformal iterated function systems : old and new results and questions*
Johannes Jaerisch	*Multifractal analysis for the geodesic flow on hyperbolic surfaces*
Dongkui Ma	*Topological entropy, topological pressure of free semigroup actions for non-compact sets and some applications*
Eugen Mihailescu	*Dynamics and dimension for a class of skew-product transformations*
Giulio Tiozzo	*The Poisson boundary of hyperbolic groups without moment conditions*
Benjamin Weiss	*Some qualitative aspects of entropy*

Contributed talks

Sergey Bezuglyi	*Endomorphisms of a measure space and representations of Cuntz relations*
Grzegorz Guzik	*Supports of evolution systems of measures for Markov semigroup*
Olena Karpel	*Dynamics on the path space of generalized Bratteli diagrams*
Grzegorz Kleszcz	*Asymptotic relation for set-valued processes*
Sebastian Kopacz	*Uniquely ergodic quasitilings of amenable groups*
Dominik Kwietniak	*Every Choquet simplex is the space of invariant probability measures of a minimal homeomorphism on a manifold*
Radu Munteanu	*Inverse measure-theoretic entropy*
Anima Nagar	*On almost periodicity and minimality for semiflows*
Jaqueline Siqueira	*On ergodic theory of impulsive semiflows*
Mariusz Urbanski	*Dimension spectrum for conformal Iterated Function Systems*
Paulo Varandas	*On the thermodynamic formalism of (semi)group actions*
Mateusz Więcek	*Asymptotic pairs in topological actions of countable amenable groups*

Program

Monday, June 26th

8:30- 9:50 *Registration*

9:50-10:00 *Opening*

10:00-10:50 **Benjamin Weiss** *Some qualitative aspects of entropy*

11:00-11:30 *Coffee break*

11:30-11:55 **Paulo Varandas** *On the thermodynamic formalism of (semi)group actions*

12:00-12:25 **Grzegorz Guzik** *Supports of evolution systems of measures for Markov semigroup*

12:30-12:55 **Sebastian Kopacz** *Uniquely ergodic quasitilings of amenable groups*

13:00-15:00 *Lunch at Arche Vita Restaurant, Matejki 11*

15:00-15:50 **Maria Carvalho** *Historic behavior within semigroup actions*

16:00-16:30 *Coffee break*

16:30-16:55 **Radu Munteanu** *Inverse measure-theoretic entropy*

17:00-17:25 **Mariusz Urbański** *Dimension spectrum for conformal Iterated Function Systems*

Tuesday, June 27th

9:00- 9:50 **Dongkui Ma** *Topological entropy, topological pressure of free semigroup actions for non-compact sets and some applications*

10:00-10:50 **Eugen Mihailescu** *Dynamics and dimension for a class of skew-product transformations*

11:00-11:30 *Coffee break*

11:30-11:55 **Sergey Bezuglyi** *Endomorphisms of a measure space and representations of Cuntz relations*

12:00-12:25 **Mateusz Więcek** *Asymptotic pairs in topological actions of countable amenable groups*

12:30-12:55 **Grzegorz Kleszcz** *Asymptotic relation for set-valued processes*

13:00-15:00 *Lunch at Arche Vita Restaurant, Matejki 11*

15:00-15:50 **Tushar Das** *Dimension theory of conformal iterated function systems : old and new results and questions*

19:00-22:00 *Banquet at Arche Vita Restaurant, Matejki 11*

Wednesday, June 28th

9:00- 9:50 **Johannes Jaerisch** *Multifractal analysis for the geodesic flow on hyperbolic surfaces*

10:00-10:50 **Udajan Darji** *Odometers and Solenoids*

11:00-11:30 *Coffee break*

11:30-11:55 **Dominik Kwietniak** *Every Choquet simplex is the space of invariant probability measures of a minimal homeomorphism on a manifold*

12:00-12:25 **Anima Nagar** *On almost periodicity and minimality for semiflows*

12:30-12:55 **Jaqueline Siqueira** *On ergodic theory of impulsive semiflows*

13:00-15:00 **Lunch** *at Arche Vita Restaurant, Matejki 11*

15:00-15:50 **Giulio Tiozzo** *The Poisson boundary of hyperbolic groups without moment conditions*

16:00-16:30 *Coffee break*

16:30-16:55 **Olena Karpel** *Dynamics on the path space of generalized Bratteli diagrams*

17:00-17:10 *Closing*

Abstracts

Endomorphisms of a measure space and representations of Cuntz relations

Sergey Bezuglyi
University of Iowa, USA

The goal of the talk is to present new classes of function systems as part of multiresolution analyses. The approach is representation theoretic, and it makes use of generalized multiresolution function systems (MRSs). As the main tool in our approach, we make precise new classes of generalized MRSs which arise directly from dynamics generated by surjective endomorphisms on measure spaces. In particular, we give the necessary and sufficient conditions for a family of functions to define generators of Cuntz relations. We find an explicit description of the set of generalized wavelet filters. The talk is based on a joint paper with Palle Jorgensen, arXiv:2304.14558.

Historic behavior within semigroup actions

Maria Carvalho
University of Porto, Portugal

In this talk, based on joint work with Vinícius Coelho (Federal University of West Bahia), Luciana Salgado (Federal University of Rio de Janeiro) and Paulo Varandas (Federal University of Bahia & CMUP), I will present a sufficient condition for a continuous transformation, acting on a Baire metric space, to exhibit a generic set of points with historic behavior (also known as irregular set). This condition turns out to be also necessary in case the map has a dense orbit, and allows a generalization which provides information on the irregular set of some semigroup actions. In particular, we extend Furstenberg's theorem on uniquely ergodic amenable group actions by homeomorphisms on a compact metric space.

Odometers and Solenoids

Udayan Darji
University of Louisville, USA

This talk will consist of two parts, one part concerning dynamics of bounded linear operators and the other concerning compact connected abelian groups. In the first part, we will show measures on odometers can lead to chaotic behavior among bounded linear operators on L^p spaces. In the second part, we will show that 1-dimensional solenoids arise as generic compact connected abelian groups.

Dimension theory of conformal iterated function systems : old and new results and questions

Tushar Das
University of Wisconsin - La Crosse, USA

I hope to present a few results and several open questions on the dimension theory of conformal iterated function systems (CIFS). The first paper (arXiv:1910.10259) considers small perturbations of a CIFS extending work of Good, Hensley and others; the second (arXiv:2007.10554) resolves some questions posed by Chousionis, Leykekhman and Urbański regarding the dimension spectrum of a CIFS (i.e. the set of all Hausdorff dimensions of its various subsystem limit sets); and the third (arXiv:2305.11829) studies the exact dimension of the prime continued fraction limit set.

Supports of evolution system of measures of Markov semigroup

Grzegorz Guzik
AGH University of Science and Technology, Poland

In 2000 it was proved by A. Lasota and J. Myjak that the support of the unique ergodic invariant measure of any homogenous asymptotically stable Markov-Feller chain on a Polish space is a semifractal set, more precisely it is the semiattractor of a Markov multifunction associated with the transition operator.

In the case of time-dependent non-homogenous Markov processes more natural counterpart of invariant measures are evolution systems of measures. We ask what is the explicit form of supports of measures in such a system in the case of so-called stronly mixing property (the equivalent of asymptotic stability). More precisely, we cosider a semigroup $(P_{t,s})_{(t,s)\in\mathbb{T}^2_\geq}$ of transition operators for non-homogenous Markov-Feller process, here $\mathbb{T}$ is a non–trivial subgroup of additive group $(\mathbb{R},+)$ of all reals and $\mathbb{T}^2_\geq := \{(t,s)\in\mathbb{T}\times\mathbb{T} : \ t\geq s\}$. Our main result reads as follow.

Theorem 1. *Assume that the semigroup $(P_{t,s})_{(t,s)\in\mathbb{T}^2_\geq}$ of Markov–Feller operators $P_{t,s} : \mathcal{M}(X)\to\mathcal{M}(X)$ on measures generated by the transition function $\{\pi_{s,t} : X\times\mathcal{B}(X)\to[0,1] : \ (t,s)\in\mathbb{T}^2_\geq\}$ with corresponding Markov multifunctions $\{\Gamma_{t,s} : X\to\mathcal{P}(X) : \ (t,s)\in\mathbb{T}^2_\geq\}$ (supports of transition probabilities) admits a strongly mixing evolution system of measures $(\nu_t)_{t\in\mathbb{T}}$. Then*

$$\operatorname{supp}\nu_t = C_\infty(t)$$

for every $t\in\mathbb{T}$, where $(C_\infty(t))_{t\in\mathbb{T}}$ with

$$C_\infty(t) = \bigcap_{s\leq t} \operatorname{cl}\Gamma_{t,s}C(s) \quad \text{for } t\in\mathbb{T}$$

is the core of an evolution semiattractor $\{C(t) : t \in \mathbb{T}\}$ *of the set–valued process* $\{\Gamma_{t,s} : (t,s) \in \mathbb{T}^2_{\geq}\}$ *given by*

$$C(t) = \bigcap_{x \in X} \liminf_{s \to -\infty} \Gamma_{t,s}(x) \quad \text{for } t \in \mathbb{T}.$$

Multifractal analysis for the geodesic flow on hyperbolic surfaces

Johannes Jaerisch
Nagoya University, Japan

We use multifractal analysis to investigate the long-term behavior of growth rates associated with the geodesic flow on surfaces of constant negative curvature. The growth rates we consider are given by the distance travelled on the surface [4, 3], the number of windings around cusps [2], and the number of crossings of sides of a fundamental domain. This talk is based on joint work with Hiroki Takahasi (Keio University).

Let G denote a finitely generated non-elementary Fuchsian group acting in the Poincaré disc model $(\mathbb{D}, d)$ of hyperbolic space. Let R be a convex, locally finite fundamental domain for G which has even corners [1, 5] and contains 0 in its interior. The finite set of side-pairings of R is denoted by G_0 and defines a symmetric set of generators of G.

Let $\mathscr{R}$ denote the set of oriented complete geodesics joining two points in $\mathbb{S}^1$ and intersecting the interior of R. If $\eta \in \mathscr{R}$ cuts through the copies $R, g_0R, g_0g_1R, \ldots$ of R, with $g_i \in G_0$ and $i = 0, 1, \ldots \in \mathbb{N}$, then $g_0, g_1, g_2, \ldots$ is called the cutting sequence of η. By slightly perturbing geodesics cutting through a vertex of R we can define for each $\eta \in \mathscr{R}$ a unique finite or infinite cutting sequence [5]. For $\eta \in \mathscr{R}$ with cutting sequence $g_0, g_1, \ldots$ of length at least $n \geq 1$, we define

$$t_n(\eta) = d(0, g_0 g_1 \cdots g_{n-1} 0),$$

and we call $t_n(\eta)/n$ the homological growth rate of η [4].

We denote by $\Lambda = \Lambda(G)$ the limit set of G, and by $\Lambda_c = \Lambda_c(G)$ the conical limit set of G. One can show that $\eta \in \mathscr{R}$ has an infinite cutting sequence if and only if its positive endpoint η^+ belongs to Λ_c. For $\alpha \geq 0$ we define the *level set*

$$\mathscr{H}(\alpha) = \left\{ \xi \in \Lambda_c : \text{ there exists } \eta \in \mathscr{R} \text{ such that } \eta^+ = \xi \text{ and } \lim_{n \to \infty} \frac{t_n(\eta)}{n} = \alpha \right\}.$$

Let $\dim_{\mathrm{H}}$ denote the Hausdorff dimension on $\mathbb{S}^1$. We call $\alpha \mapsto \dim_{\mathrm{H}} \mathscr{H}(\alpha)$ the $\mathscr{H}$-spectrum. The range of the spectrum is given by

$$\underline{\alpha} = \inf_{g \in G \setminus \{1\}} \frac{d(0, g0)}{|g|} \quad \text{and} \quad \overline{\alpha} = \sup_{g \in G} \frac{d(0, g0)}{|g|},$$

where $|g|$ denotes the word length of $g \in G$.

Theorem 2. *The* $\mathscr{H}$*-spectrum is continuous on* $[\underline{\alpha}, \overline{\alpha}]$*, analytic on* $(\underline{\alpha}, \overline{\alpha})$.

From now we shall assume that R has cusps and that the sides of R are pairwise disjoint. The set of parabolic generators and the set of hyperbolic generators are respectively denoted by

$$\Gamma_0 = \{\gamma_i^{\pm 1} \in G_0 : \ 1 \le i \le m\} \ \text{ and } \ H_0 = G_0 \setminus \Gamma_0.$$

The cutting sequence $\omega = g_0, g_1, g_2, \ldots$ of an element of $\mathscr{R}$ with positive endpoint $x \in \Lambda_c(G)$ is not eventually constant to some element of Γ_0. We can therefore decompose ω into a sequence of blocks $(B_j(x))_{j\ge 1}$, where each hyperbolic generator in ω forms a block of length one. For parabolic generators in ω we build maximal blocks of consecutive appearances of the same parabolic generator. We have either $B_j(x) = h$ for some $h \in H_0$, or $B_j(x) = \gamma^{\ell+1}$ for some $\gamma \in \Gamma_0$ and $\ell \ge 0$.

We introduce the *multi-cusp winding process* $(a_{i,j})_{1\le i\le m, 1\le j}$ given by

$$a_{i,j}(x) = \begin{cases} \ell & \text{if } B_j(x) = \gamma_i^{\pm(\ell+1)}, \quad \ell \ge 0 \\ 0 & \text{otherwise.} \end{cases}$$

A block $\gamma^{\ell+1}$ in the cutting sequence ω, for some parabolic generator $\gamma \in \Gamma_0$ and $\ell \ge 0$, means that the geodesic spirals ℓ times around the cusp associated with γ. Let

$$D_m = \{\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_m) \in \mathbb{R}^m : \ \alpha_i \ge 0 \text{ for all } 1 \le i \le m\}.$$

For $\boldsymbol{\alpha} \in D_m$ we define

$$B(\boldsymbol{\alpha}) = \left\{ x \in \Lambda_c(G) : \ \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} a_{i,j}(x) = \alpha_i \text{ for all } 1 \le i \le m \right\}.$$

The Bowen-Series map [1] $f : \Delta \to \mathbb{S}^1$ is defined on a subset $\Delta \subset \mathbb{S}^1$ containing $\Lambda(G)$ and is piecewise analytic. We also define $\tau : \Lambda_c(G) \to \mathbb{N}$, where $\tau(x)$ is given by the length of the block $B_1(x)$. The induced Bowen-Series map is defined by $\tilde{f} : \Lambda_c(G) \to \Lambda_c(G)$, $\tilde{f}(x) = f^{\tau(x)}(x)$. We denote by $\mathcal{M}(\tilde{f})$ the set of $\tilde{f}$-invariant Borel probability measures on $\Lambda_c(G)$ with positive and finite Lyapunov exponent $\chi(\mu) = \int \log|(\tilde{f})'| d\mu$, where the norm is taken with respect to the Euclidean metric. For $\mu \in \mathcal{M}(\tilde{f})$ we denote by $h(\mu)$ the Kolmogoroff-Sinai entropy of $\tilde{f}$ with respect to μ.

Theorem 3. *For all $\boldsymbol{\alpha} \in D_m$ we have $B(\boldsymbol{\alpha}) \neq \emptyset$ and*

$$\dim_H B(\boldsymbol{\alpha}) = \limsup_{\epsilon\to 0} \left\{ \frac{h(\mu)}{\chi(\mu)} : \ \mu \in \mathcal{M}(\tilde{f}), \ \left| \int a_{i,1}\, d\mu - \alpha_i \right| < \epsilon \text{ for all } 1 \le i \le m \right\}.$$

References

[1] R. Bowen, C. Series, Markov maps associated with Fuchsian groups. Inst. Hautes Études Sci. Publ. Math. **50** (1979) 153–170.

[2] J. Jaerisch, H. Takahasi: Mixed multifractal spectra of Birkhoff averages for non-uniformly expanding one-dimensional Markov maps with countably many branches. Advances in Mathematics **385** (2021).

[3] J. Jaerisch, H. Takahasi: Multifractal analysis of homological growth rates for hyperbolic surfaces. Preprint Arxiv (2022).

[4] Kesseböhmer, M., Stratmann, B. O.: A Multifractal formalism for growth rates and applications to geometrically finite Kleinian groups. Ergodic Theory and Dynamical Systems **24** (2004) 141–170.

[5] Series, C.: Geometrical Markov coding of geodesics on surfaces of constant negative curvature. Ergodic Theory and Dynamical Systems **6** (1986) 601–625.

Dynamics on the path space of generalized Bratteli diagrams

Olena Karpel
AGH University of Science and Technology, Poland

Bratteli diagrams are a powerful tool for the study of dynamial systems in measurable, Cantor and Borel dynamis. The set of invariant measures, minimal components, structure of the orbits of the transformation become more transparent when one deals with the corresponding Bratteli-Vershik dynamical systems. We consider various classes of generalized Bratteli diagrams which appear in Borel dynamics, study the properties of their tail equivalence relations and the corresponding Vershik maps, describe the sets of invariant finite and infinite σ-finite Borel measures. We emphasize differences (and similarities) between generalized and classical Bratteli diagrams. The talk is based on a joint work with Sergey Bezuglyi, Palle E.T. Jorgensen and Shrey Sanadhya. The work is supported by the NCN (National Science Center, Poland) Grant 2019/35/D/ST1/01375 and the program "Excellence initiative - research university" for the AGH University of Science and Technology.

Asymptotic relation for set-valued processes

Grzegorz Kleszcz
AGH University of Krakow, Poland

Consider a set of moments of time $\mathcal{T} \subset \mathbb{R}$ as a subgroup of $(\mathbb{R}, +)$. A semi-group

$$(F_{s,t})_{(s,t)}, \quad s \geq t, \quad s,t \in \mathcal{T},$$

with composition

$$F_{s,r} \circ F_{r,t} = F_{s,t}, \quad s \geq r \geq t, \quad s,t,r \in \mathcal{T},$$

where the family $F_{s,t} : X_t \multimap X_s$ of lower semicontinuous multifunctions is defined on a family of topological spaces X_t, $t \in \mathcal{T}$.

We compare non-autonomous attractors of this semi-group with Conley attractor of some limit relation build up from original semi-group. The results are generalisation of [1].

The talk is based on joint work with Grzegorz Guzik.

References

[1] Manjunath, G. and Jaeger, H., The Dynamics of Random Difference Equations Is Remodeled by Closed Relations , SIAM Journal

Uniquely ergodic quasitiling of amenable groups

Sebastian Kopacz
Wroclaw University of Science and Technology, Poland

We prove that if G is a countable amenable group then there exists a disjoint uniquely ergodic quasitiling, i.e., if we view a quasitiling as an element of a symbolic dynamical system then the closed orbit of this element supports only one ergodic measure. We achieve this by starting with some quasitiling system and inductively modifying it to gradually decrease the diameter of the corresponding set of invariant measures.

This is a joint work with Dawid Huczek (WUST) and Jacek Serafin (WUST).

Every Choquet simplex is the space of invariant probability measures of a minimal homeomorphism on a manifold

Dominik Kwietniak
Jagiellonian University, Poland

The realisation problem is one of the most intriguing questions in the dynamical systems theory: given a property P that a dynamical system may or may not have, and a family C of dynamical systems decide whether a system exhibiting P exists in C. A very famous instance of that question is the smooth realisation problem. It asks if every measure preserving system isomorphic to a system given by a strictly ergodic smooth diffeomorphism of a compact manifold preserving a measure equivalent to the volume element. The question is inspired by a brief remark made in 1932 by Johnny von Neumann in his foundational paper on ergodic theory.

During my talk, I will discuss the recent progress on the following variant of the realisation problem: which Choquet simplices can be realised as spaces of invariant measures of minimal homeomorphisms on manifolds? Our main result says that for every manifold carrying a strictly ergodic homeomorphism if K is a Choquet simplex, then K is affinely homeomorphic to the space of invariant probability measures of some minimal homeomorphism on that manifold (joint work with Sejal Babel, Jernej Činč, Till Hauser, Piotr Oprocha).

Topological entropy, topological pressure of free semigroup actions for non-compact sets and some applications

Dongkui Ma
South China University of Technology, China

In this talk, we adopt the Carathéodory-Pesin structure(C-P structure) and introduce the notions of the topological entropy (pressure) and lower and upper capacity topological entro-

pies (pressure) of a free semigroup action for an arbitrary subset. We provide some properties of these notions. As application s, we give some estimations of the topological entropy (pressure) or upper capacity topological entropy of a free semigroup action on certain non-compact sets. By using the Bowen's equation with respect to the topological pressure, we characterize the Hausdorff dimension of an arbitrary subset, where the points of the subset have the positive lower Lyapunov exponents and satisfy a tempered contraction condition. Our analysis generalizes the results obtained by Bufetov, Misiurewicz, Ma-Wen, Huang, Tian, Chen, Lau and Climenhaga et al..

Dynamics and dimension for a class of skew-product transformations

Eugen Mihailescu
Romanian Academy, Romania

We study dynamics for a class of skew-product transformations which are only piecewise differentiable on countably many pieces and non-conformal. There exist random countably generated fractal limit sets obtained from iterations in the fibers of these maps. We prove a global volume lemma for the push-forward of equilibrium measures from a shift space to the global basic set. A dimension formula of Ledrappier-Young type is then obtained for these global measures. We study next the geometric potentials ψ_s, and prove that the dimensions of the associated invariant measures in fibers depend real-analytically on the parameter. A Variational Principle for dimension is also established.

Inverse measure-theoretic entropy

Radu Munteanu
University of Bucharest, Romania

We introduce a notion of inverse entropy for an arbitrary measure preserving endomorphism on a metric space. We obtain a general formula for inverse measure - theoretic entropy relating it to the folding entropy. We compute the inverse entropy for some concrete examples. The inverse entropy can differentiate between endomorphisms which have the same forward measure theoretic entropy. We can also compute the inverse entropy for the SRB measure of fat baker's transformations.

This is joint work with Eugen Mihailescu.

On almost periodicity and minimality for semiflows

Anima Nagar
Indian Institute of Technology, India

In topological dynamics, the dynamical behavior sometimes has a sharp contrast when the action is by semigroups or monoids to when the action is by groups. In this short note we bring out this contrast while discussing the equivalence of almost periodicity and minimality, and some implications when every point is an almost periodic point.

This is a joint work with Joseph Auslander.

On ergodic theory of impulsive semiflows

Jaqueline Siqueira
University of Porto, Portugal & Federal University of Rio de Janeiro, Brasil

Impulsive Dynamical Systems (IDS) can be seen as suitable mathematical models of real world phenomena that display abrupt changes in their behaviour.

More precisely, an IDS is described by three objects: a continuous semiflow on a space X; a set D contained in X where the flow undergoes sudden perturbations; and an impulsive function from D to X, which determines the change in the trajectory each time it collides with the impulsive set D.

In spite of their great range of applications, IDS have started being studied from the viewpoint of ergodic theory only quite recently in the work of Alves and Carvalho [2014]. A key challenge, inherent to the dynamics, is that in general, an impulsive semiflow is not continuous.

In this talk I will provide sufficient conditions for the existence of invariant measures that imply the ones given by Alves and Carvalho and are somewhat easier to verify. Moreover, I will discuss how typical is the invariance of the non-wandering set of an impulsive semiflow. I will finish the talk with some open problems. This talk is based upon two works with Afonso and Bonotto and with Torres and Varandas

---

The Poisson boundary of hyperbolic groups without moment conditions

Giulio Tiozzo
University of Toronto, Canada

The Poisson boundary is a measure-theoretic object attached to a group equipped with a probability measure, and is closely related to the notion of harmonic function on the group. In many cases, the group is also endowed with a topological boundary arising from its geometric structure, and a recurring research theme is to discuss the relation between the two notions of boundary.

In this talk, we prove that the Poisson boundary of a random walk with finite entropy on a non-elementary hyperbolic group can be identified with its hyperbolic boundary, without assuming any moment condition on the measure. In this generality, this identification result is new even for free groups. We will then discuss extensions of this result to other groups with hyperbolic properties.

Joint with K. Chawla, B. Forghani, and J. Frisch.

Dimension spectrum for conformal Iterated Function Systems

Mariusz Urbański
University of North Texas, USA

I will define conformal iterated function systems S over a countable alphabet E and their limit sets (attractors) J_E. I will discuss the formula for the Hausdorff dimension of this limit set, commonly referred to as a version of Bowen's formula, involving topological pressure. The main focus will be on the set

$$Sp(E) = \{HD(J_F) : F \subset E\},$$

called the dimension spectrum of the system S. I will prove that always

$$Sp(E) \supset (0, \theta_E),$$

where θ_E is the finiteness parameter of S (will be defined). I will also construct a system for which $Sp(E)$ is a proper subset of $(0, HD(J_E)]$. I will then discuss the property that

$$Sp(S) = (0, HD(J_E)],$$

called the full spectrum dimension property. In particular, I will discuss the conformal iterated function systems and their various subsystems, generated by real and complex continued fraction algorithms, and will show that many of them (subsystems) enjoy the full spectrum dimension property.

On the thermodynamic formalism of (semi)group actions

Paulo Varandas
Federal University of Bahia, Brazil & University of Porto, Portugal

The thermodynamic formalism in dynamical systems aims to relate topological and measure theoretical notions of complexity by means of variational principles, and to characterize, whenever possible, the probability measures attaining the maximal topological complexity. Focusing on the context of finitely generated semigroup actions, I will talk on how convex analysis can be used to define entropy-type functions and to obtain a unified thermodynamic formalism for this class of semigroup actions.

This is based on joint works with A. Biś, M. Carvalho, M. Mendes and G. Pessil.

Some qualitative aspects of entropy

Benjamin Weiss
Hebrew University of Jerusalem, Israel

I will briefly describe an old result which gives a characterization of the entropy of ergodic processes as the unique numerical isomorphism invariant satisfying a certain natural condition. I will then discuss a rather surprising recent result in which the dichotomy between positive and zero entropy is characterized in terms of generic extensions of ergodic systems.

This represents joint work with D. Ornstein, T. Austin, E. Glasner and J-P. Thouvenot.

Asymptotic pairs in topological actions of countable amenable groups

Mateusz Więcek
Wroclaw University of Science and Technology, Poland

By the known theorem of F. Blanchard, B. Host and S. Ruette, every topological $\mathbb{Z}$-action of positive entropy admits asymptotic pairs. Moreover, T. Downarowicz and Y. Lacroix proved that every topological $\mathbb{Z}$-action of entropy zero has an extension with no asymptotic pairs. Together, these two results give a characterization of zero-entropy topological $\mathbb{Z}$-actions as factor of systems with no asymptotic pairs. Recently developed theory of multiorders allowed us to achieve a similar characterization for topological actions of countable amenable groups.

In the talk, we introduce a definition of a $\prec$-asymptotic pair in a topological action of a countable group G, where $\prec$ is an order on G of type $\mathbb{Z}$. We then provide a theorem stating that if G is a countable amenable group and (X,G) is a topological G-action of positive entropy, then for every multiorder $(\tilde{\mathcal{O}},\nu,G)$ and ν-almost every order $\prec\in\tilde{\mathcal{O}}$ there exists a $\prec$-asympotic pair in X. This result is a generalization of the Blanchard-Host-Ruette Theorem. We will also show that for every countable amenable group G, and every multiorder on G arising from a tiling system, every topological G-action of entropy zero has an extension which has no $\prec$-asymptotic pairs for any $\prec$ belonging to this multiorder. Therefore, we have the following characterization of topological G-actions of entropy zero: (X,G) has topological entropy zero if and only if there exists a multiorder $(\tilde{\mathcal{O}},\nu,G)$ on G and an extension (Y,G) of (X,G), such that for ν-almost any $\prec\in\tilde{\mathcal{O}}$, there are no $\prec$-asymptotic pairs in (Y,G).

The talk is based on the joint work with Tomasz Downarowicz.

List of participants

1.	**Sergey Bezugly**	University of Iowa, USA
2.	**Andrzej Biś**	University of Lodz, Poland
4.	**Henk Bruin**	University of Vienna, Austria
5.	**Zoltán Buczolich**	ELTE Eötvös Loránd University, Hungary
6.	**Maria Carvalho**	University of Porto, Portugal
7.	**Udayan Darji**	University of Louisville, USA
8.	**Tushar Das**	University of Wisconsin - La Crosse, USA
9.	**Dikran Dikranyan**	Udine University, Italy
10.	**Tomasz Downarowicz**	Wroclaw University of Science and Technology, Poland
11.	**Małgorzata Filipczak**	University of Lodz, Poland
12.	**Magdalena Foryś-Krawiec**	AGH University of Krakow, Poland
13.	**Bartosz Frej**	Wroclaw University Of Science And Technology, Poland
14.	**Grzegorz Guzik**	AGH University of Science and Technology, Poland
15.	**Roman Hric**	Matej Bel University, Slovakia
16.	**Grażyna Horbaczewska**	University of Lodz, Poland
17.	**Johannes Jaerisch**	Nagoya University, Japan
18.	**Olena Karpel**	AGH University of Science and Technology, Poland
19.	**Grzegorz Kleszcz**	AGH University of Krakow, Poland
20.	**Sebastian Kopacz**	Wrocław University of Science and Technology, Poland
21.	**Dominik Kwietniak**	Jagiellonian University, Poland
22.	**Dongkui Ma**	South China University of Technology, China
23.	**Agnieszka Marczuk**	University of Lodz, Poland
24.	**Eugen Mihailescu**	Romanian Academy, Romania
25.	**Michaela Mihoková**	Matej Bel University, Slovakia
26.	**Radu Munteanu**	University of Bucharest, Romania
27.	**Anima Nagar**	Indian Institute of Technology, India
28.	**Ryszard Pawlak**	University of Lodz, Poland
29.	**Michał Prusik**	Wroclaw University Of Science And Technology, Poland
30.	**Silvia Radinger**	University of Vienna, Austria
31.	**Lenka Rucka**	Silesian University in Opava, Czech Republic
32.	**Klaus Schmidt**	University of Viennal Austria
33.	**Jacek Serafin**	Wroclaw University of Science and Technology, Poland
34.	**Jaqueline Siqueira**	University of Porto, Portugal & Federal University of Rio de Janeiro, Brasil
35.	**L'ubomír Snoha**	Matej Bel University, Slovakia
36.	**Hiroki Sumi**	Kyoto University, Japan
37.	**Giulio Tiozzo**	University of Toronto, Canada
38.	**Mariusz Urbański**	University of North Texas, USA
39.	**Paulo Varandas**	Federal University of Bahia, Brazil & University of Porto, Portugal
40.	**Benjamin Weiss**	Hebrew University of Jerusalem, Israel
41.	**Mateusz Więcek**	Wroclaw University of Science and Technology, Poland